# Guidance of the aircraft with model-based predictive controller with position-based dynamic reference

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*Abstract*— Fixed-wings aircraft maneuvers in the terminal zone of the airports have recently been augmented with vision sensors in order to enable completely autonomous operations of the aircraft. The task of the JSI group in the feasibility study of autonomous vision aided maneuvers is to apply an advanced controller to the fixed-wings aircraft and highlight the advantages of use of such a controller in combination with position estimated from the outputs of the vision sensors. Because of its ability to take into account future signals and the constraints on these signals, model predictive control has been chosen.

#### I. INTRODUCTION

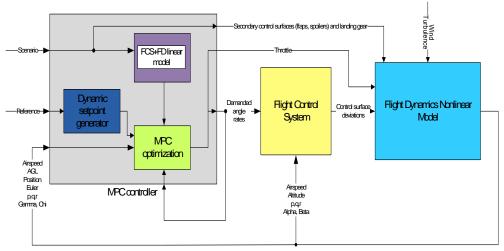
THE purpose of the PEGASE (helicoPter and aEronef naviGation Airborne SystEms) project [1] is to prepare the development of an autonomous (no external assistance or ground equipment required), all weather conditions, localization and guidance system based upon correlation between vision sensors output and a ground reference database.

The ambitions of the PEGASE project are twofold. Firstly, to achieve a cost effective navigation means that has higher accuracy and integrity than existing ones, yet is not susceptible to jamming. Secondly, it can help reduce noise levels and fuel consumption through new procedures in the terminal zone and address flow delays in adverse weather conditions.

The task of the JSI group in the PEGASE project is to implement an advanced controller, which would be able to guide the fixed-wings aircraft in all visual and weather conditions and to highlight the advantages of use of such a controller. Two techniques for combining vision sensors output with advanced control have been proposed: position based visual servoing (PBVS) (the signals derived from the image are aircraft positions) and image based visual software for estimating the aircraft position from the image has been made by our partners in the project. These partners are: research groups at INRIA Lagadic, INRIA Vista, INRIA Sophia, CNIT, CNRS and EPFL.

Because of the ability to take into account future values of the signals and the constraints on them model-based predictive control (MPC) [3] has been chosen. Since the reference trajectories in aerospace are normally position based, a link between time profiles and spatial coordinates had to be established. For that purpose we have designed a dynamic reference generator, which in each sampling instant generates a sequence of positions, angles and airspeeds, that represents a reference for the controller in that time instant. This sequence is determined on basis of current position and airspeed and therefore the needed link between time and space control. If aircraft is already on the demanded path, then all the points in the sequence of positions are located along the reference trajectory. On the other hand, if the aircraft current position is away from or near the demanded path, the sequence of reference positions starts in the current aircraft position and then smoothly converges to the reference trajectory.

This paper is set out as follows. In section 2 we present the control scheme and its elements. In section 3 the position based model predictive control algorithm is described. Section 4 provides results whereas conclusions are given in section 5.



servoing (IBVS) (the signals derived from the image are errors in angles and positions between features in reference and actual image). In this paper we focus on the PBVS. The

Fig 1 The control scheme of the PEGASE project.

## II. CONTROL SCHEME

Figure 1 displays the block scheme of the control part of the PEGASE project. The controlled plant is a combination of the flight model and flight control system. In the gray box there is a structure of the controller.

Let us first take a look at the controlled plant blocks, the flight dynamic model and the flight control system, before we concentrate on the main part, which is the MPC controller.

## A. Flight Dynamic model

The MATLAB Simulink block diagram of the flight dynamic model is presented in figure 2. Equations of motion are used to derive aircraft state (velocities in body, position in earth axes, Euler angles and angle rates in body axes) from forces and moments, elatve to center of gravity, expressed in body axes (Fx, Fy, Fz, L, M, N). States are computed using the following equations:

Aircraft velocities in body axes  $v_x$ ,  $v_y$  and  $v_z$ :

$$\dot{v}_x = \frac{F_x}{m} + rv_y - qv_z \tag{1}$$

$$\dot{v}_y = \frac{F_y}{m} + pv_z - rv_x \tag{2}$$

$$\dot{v}_z = \frac{F_z}{m} + qv_x - pv_y \tag{3}$$

,where  $F_x$ ,  $F_y$  and  $F_z$  are forces in body axes, *m* is the mass of the aircraft, and *p*,*q* and *r* are the angle rates in body axes.

The positions in North-East-Down coordinate system:

$$\begin{bmatrix} \dot{x}_{N} \\ \dot{x}_{E} \\ \dot{x}_{D} \end{bmatrix} = M_{BE} \begin{bmatrix} v_{x} \\ v_{y} \\ v_{z} \end{bmatrix}$$
(4)

Angle rates in body axes:

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = I^{-1} \begin{bmatrix} L - I_{zz}qr + I_{yy}qr \\ M - I_{xx}pr + I_{zz}pr \\ N - I_{yy}pq + I_{xx}pq \end{bmatrix}$$
(5)

,where L,M and N are the moments on body axes.

Euler angles:

$$\dot{\varphi} = p + q \sin \varphi \tan \vartheta + r \cos \varphi \tan \vartheta$$
(6)

$$\mathcal{G} = q\cos\varphi - r\sin\varphi \tag{7}$$

$$\dot{\psi} = q \frac{\sin\varphi}{\cos\vartheta} + r \frac{\cos\varphi}{\cos\vartheta} \tag{8}$$

,where  $\phi$  is the roll angle,  $\theta$  is the pitch angle and  $\psi$  is the yaw angle.

Rotational and interactions matrix needed in equations (4) and (5) are computed with equations (9) and (10).

$$M_{BE} = \begin{bmatrix} \cos\psi\cos\theta & \sin\psi\cos\theta & -\sin\theta\\ \cos\psi\sin\theta\sin\phi & \sin\psi\cos\phi & \sin\psi\sin\theta\sin\phi & \cos\phi\cos\phi & \cos\theta\sin\phi\\ \cos\psi\sin\theta\cos\phi & \sin\psi\sin\phi & \sin\psi\sin\theta\cos\phi & \cos\psi\sin\phi & \cos\phi\cos\phi \end{bmatrix}$$
(9)  
$$I = \begin{bmatrix} I_{xx} & 0 & 0\\ 0 & I_{yy} & 0\\ 0 & 0 & I_{zz} \end{bmatrix}; I^{-1} = \begin{bmatrix} 1/I_{xx} & 0 & 0\\ 0 & 1/I_{yy} & 0\\ 0 & 0 & 1/I_{zz} \end{bmatrix}$$
(10)

Total airspeeds *u*, *v* and *w* along body axes are:

$$u = v_x - u_w; \quad v = v_y - v_w; \quad w = v_z - w_w$$
 (11)

where  $u_w$ ,  $v_w$  and  $w_w$  are wind velocities components along body axis. Total airspeed and the incidence angles are then calculated as follows:

$$V = \sqrt{u^{2} + v^{2} + w^{2}}$$
  

$$\alpha = \arctan(w/u)$$
  

$$\beta = \arcsin(v/V)$$
(12)

,where V is total airspeed,  $\alpha$  is the angle of attack and  $\beta$  is the sideslip angle. Total forces and moments are the sums of different aerodynamic gravity and engine contributions:

$$F_{x} = F_{x_{AERO}} + F_{x_{GRAV}} + F_{x_{ENG}}$$
(13)

$$F_{y} = F_{y_{AERO}} + F_{y_{GRAV}} + F_{y_{ENG}}$$
(14)

$$F_z = F_{z_{AERO}} + F_{z_{GRAV}} + F_{z_{ENG}}$$
(15)

$$L = L_{AERO} + L_{ENG} \tag{16}$$

$$M = M_{AERO} + M_{ENG} \tag{17}$$

$$N = N_{AERO} + N_{ENG} \tag{18}$$

The detailed computations are, due to lack of space, not given in this paper.

The aircraft model has 7 inputs and 13 outputs. Inputs: Primary control surfaces (1), Secondary control surfaces (2), left and right engine throttle (3 and 4), landing gear configuration (5), altitude above ground level (6), wind velocities in NED axes (7). Outputs: airspeed (1), angle of attack and sideslip angle (2), angular rates (3), Euler angles (4), position in NED coordinates (5), velocities in NED coordinates (6), velocities in body axes (7), accelerations in body axes (8), track and slope angle (9), control surface deflections (10), engine thrust (11), Mach number (12), forces of friction of landing gear (13), vind velocities in body axes (14)

# B. Flight Control System

Flight control system, depicted in figure 2, generates surface commands for ailerons, elevators and rudder given angular speeds p, q and angle of sideslip  $\beta$ . The control law is a matrix of Linear Quadratic Regulator (LQR) gains optimized for different altitudes. The states used for the control law are angle of attack  $\alpha$ , sideslip angle  $\beta$  and angular speeds p, q and r.

## III. POSITION BASED MODEL PREDICTIVE CONTROL ALGORITHM

The block diagram of the model predictive controller is given in figure 3. The algorithm is composed of 2 sub algorithms, the dynamic reference generation and the model based predictive control.

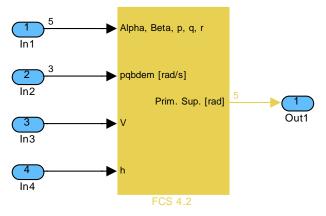


Fig 2 Simulink diagram of flight control system. Inputs: angle of attack, sideslip angle, angular rates (1), demanded roll rate, demanded pitch rate and demanded sideslip angle (2), airspeed (3), altitude (4). Outputs: primary control surfaces (1)

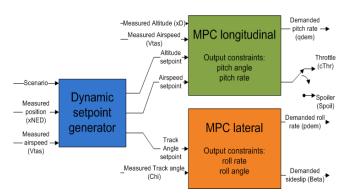


Fig 3 Position based model predictive controller with dynamic reference generator, longitudinal controller and lateral controller

#### A. Dynamic reference generation

The independent variable in the optimization of the model predictive control is time, but the reference trajectory in aerospace control is always position based. The main purpose for introducing the dynamic reference generator (blue box in figure 3) is to establish a link between position based and time based control.

With the addition of the dynamic reference generator to the model predictive controller we get a so called hybrid between time based and position based control. In each control sample instant the dynamic reference generator takes as inputs the measured position and airspeed and the prescribed scenario (composed of 3 dimensional waypoints and desired airspeeds) and generates three sequences of reference signals. These sequences are reference altitude, reference track angle (heading) and reference airspeed, respectively but are only used (for finding the optimal controller output) in the current time sample, after that another sequence is generated. Taking into account that the sample time is relatively small, we can expect a relatively small change in consequent sequences.

## 1) Reference altitude

Sequence of reference altitudes starts from the altitude of the current position of the aircraft and then smoothly converges to the demanded path (prescribed by scenario) as shown on figure 4. When far away from reference trajectory, the approach angle is constant up to a certain predetermined distance. After that the angle starts to decrease exponentially towards zero.

# 2) Reference track angle

In the lateral plain of the controller we prescribe the future reference track angles rather than future desired positions. The procedure is shown on figure 5.

#### 3) Reference airspeed

Sequence of future reference airspeeds starts in current aircraft airspeed and ends in airspeed prescribed by the scenario as shown in figure 6

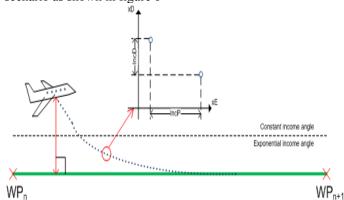


Fig 4 Altitude reference. The green line represents the prescribed trajectory defined by the scenario. The sequence is as long as the prediction horizon of the model predictive controller.

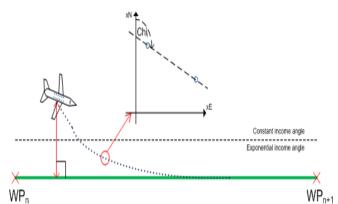
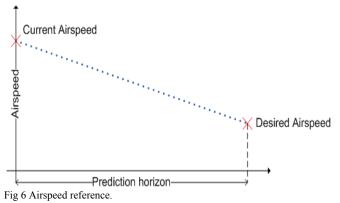


Fig 5 Track angle reference. The green line represents the prescribed trajectory defined by the scenario. The sequence is as long as the prediction horizon of the model predictive controller.



This sequence is in each control time instant fed to the model based predictive controller described in the next subsection.

#### B. Model predictive control

The basic theory behind model predictive control can be found in [3]. The control algorithm is divided into longitudinal and lateral plain as shown on figure 3. Sampling time of both controllers is 0.2 seconds and the prediction horizon is 50 samples. Both controllers use blocking strategy, which means that the manipulated variables are held constant over multiple successive sampling periods. Both controllers also use the so called *Look ahead*, the function that enables the exploitation of future references. With this function the fixed wings aircraft can "see" the coming turns, and begin the turn procedure before the actual turn commences, resulting in smaller or no overshoots and generally better tracking.

The implementation has been done using MATLAB Model Predictive Control Toolbox [6]. All the parameters of the controller are presented in tables 1 and 2. Throttle and spoiler command are designed in a split range control fashion. If the value of the second output of the longitudinal controller is positive it influences throttle input on the flight dynamic model and if it is negative values it influences on spoiler input on flight dynamic model.

### IV. RESULTS

The scenario of landing on the Marignane airport in Marseille, Provence has been tested. The reference trajectory is given with 3 dimensional waypoints, with prescribed airspeeds and fixed-aircraft configurations for each segment. between two waypoints. These waypoints and configurations are given in table 3. The results are shown in figures 7 to 11.

TABLE I LONGITUDINAL MODEL PREDICTIVE CONTROLLER

LONG	TUDINAL MO	DLLIKI	DICTIV	LCONTRO	JEEEK
Basic					
parameters					
Sample time	0.2 sec				
(Ts)					
Prediction	50				
horizon					
Control	blocking				
horizon	[10 40]				
Input	Туре	Weig		Constra	
variables	••	ht		ints	
Vtas	Measured	0.3		none	
Theta	Measured	0		-	
				20°20	
				0	
xD	Measured	1.0		none	
q	Measured	0		-0.4	
				0.4	
				rad/s	
Gamma	Unmeasure	0		none	
	d				
Output	Туре	Weig	Rate	Constra	Rate
variables		ht	weig	ints	constraint
(commands)			ht		S
q dem	Manipulate	0	10	-	none
-	d			0.40.	
				4 rad/s	
Throttle /	Manipulate	0	5	-601	none
Spoil	d				
· ·					

TABLE II ATERAL MODEL PREDICTIVE CONTROLLER

LATERAL MODEL PREDICTIVE CONTROLLER					
Basic					
parameters					
Sample time	0.2 sec				
(Ts)					
Prediction	50				
horizon					
Control	blocking				
horizon	[10 40]				
Input	Туре	Weig		Limits	
variables		ht			
р	Measured	0		-	
-				0.40.	
				4 rad/s	
r	Measured	0		-	
				0.40.	
				4 rad/s	
Phi	Measured	0		-	
				3030	
				deg.	
Psi	Measured	0		none	
Chi	Measured	1.0		none	

Output variables (commands)	Туре	Weig ht	Rate weig ht	Limits	Rate limits
p_dem	Manipulate d	0	0.1	- 0.40. 4 rad/s	none
B_dem	Manipulate d	0.3	0.1		

TABLE III MARSEILLE AIRPORT LANDING SCENARIO							
Waypo int designa tion	North Positio n	East position	Down position	Airspeed	Flaps	Lan ding gear	
IAF AVN	63331	-38278	-2133.6	113	0	0	
	47135	-31375	-1524	102.88	0	0	
MAZE T	33220	-26067	-1524	102.88	20	0	
ZEBR A	21076	-21262	-1066.8	61.73	40	1	
FAF 13L	15297	-15397	-1066.8	61.73	40	1	
THR	1599	-1610	0	61.73	40	1	
DTHR 31L	0	0	0	0	0	1	

We can see that the reference tracking is good, even in presence of noise and strong winds, but the controller is still not sufficiently robust in certain segments. The main advantage of model predictive controller is depicted in a close up of bird perspective of the aircraft trajectory in figure 11, where we can clearly observe a preliminary action of the controller, a direct consequence of the long predictive horizon and the look ahead function.

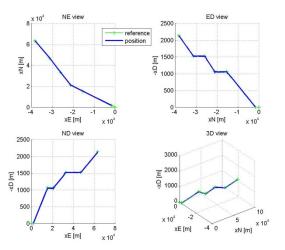


Fig 7 Up left: bird (North East - xN/xE) view of trajectory, up right: Down East (xD/xE) side view, down left: Down North (xD/xN) side view, down right: 3D view

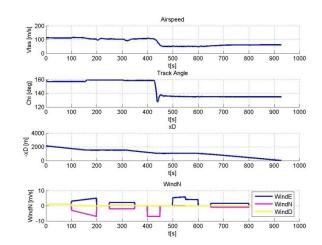


Fig 8 Controlled variables: airspeed, track angle, altitude, wind velocities in NED axes

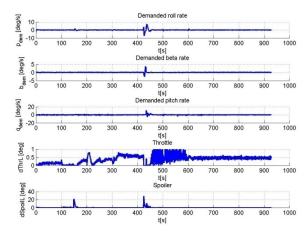


Fig 9 Controller outputs influencing on primary control surfaces: demanded roll rate, demanded sideslip angle, demanded pitch rate, throttle command, spoiler command

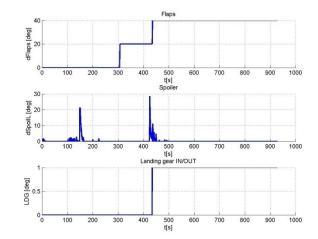


Fig 2 Flaps, spoiler command, landing gear in/out

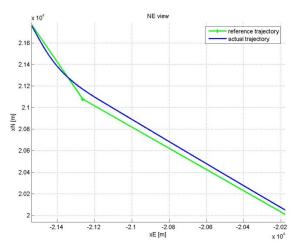


Fig 3 Close-up of the bird perspective. In this figure we can observe a preliminary reaction of the model predictive controller. (The aircraft is travelling from top to down)

# V. CONCLUSIONS

With the introduction of the dynamic reference generator we have been able to translate the model based predictive control algorithm into the spatial mode, meaning that the reference vectors fed to the controllers are dependent on the current aircraft position. This enables the plane to land on a specific location rather than at specific time, which is very useful for obvious reasons. The look ahead function allows the controller to see the future reference and thus enables the preliminary action of the controller. The controller still has some problems, specifically it the ratio between robustness and performance is not optimal at certain altitudes, speeds and configurations. Also some details, such as flare maneuvers, decrab maneuvers, have not yet been added

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